



Black-Scholes for Dividend Paying Assets

Our previous derivation of the Black-Scholes equation assumes assets to be non-dividend paying throughout their ownership. In reality assets do pay dividends,making them attractive investments. The price of the asset is largely reflectant of future and past payouts. A dividend is a share of the operating profits paid out to the shareholders at regular intervals. If an organisation fails to make a profit, then no dividend payout out is make for that period. The frequency and dates of the dividend payments vary from one organisation to another. Dividends must be considered in pricing derivatives on dividend paying assets (e.g. shares in a company) since after each dividend payment the price of the asset drops by the dividend amount. Dividend forcasting is an art in itself and very crucial to pricing derivatives. When including dividend payments in the Black-Scholes treatment we must ask ourselves two question:

1) How large are the payments?
2) At what date and how often are they paid out?

The second of these two points opens the door to a more interesting question from a mathematical perspective. Does one use a continous or discrete approach?

For practical reasons,most dividend payments are discretised to set intervals. If the intervals are small enough, one can approach a continous limit. The closest example to a continous dividend paying asset is money earnt on an interest bearing account.

Assuming our asset to have continous dividend payments, we define the dividend yield D_0 as the proportion of asset price S paid out per unit time. Therefore, in time dt,a payment of $D_0 S dt$ is made. The random walk for the asset price becomes

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D_0 is a constant in this case. This will alter our Black-Scholes PDE to

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - D_0) S \frac{\partial V}{\partial S} - rV = 0.$$

Our final conditions remain unchanged,

$$C(S, T) = \max(S - E, 0)$$

for a call option,and

$$P(S, T) = \max(E - S, 0)$$

for a put option.

:: Collaborations ::

Boundary conditions for a call option become

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as before,and

$$C(S,\ t) \sim S\ \exp(-D_0\ (T-t))$$

as $S \to \infty$.

Note: if [(, then we recover our previous boundary condition where the option is worth the asset price.

Similarly, the boundary conditions for a put option on a dividend paying asset are

$$P(0,\ t) = E\ \exp((r-D_0)\ (T-t))$$

and

$$P(S,\ t) = 0$$

as $S \to \infty$.

The Black-Scholes equation for a dividend paying asset is solved the same way as the non-dividend paying one, replacing r by [(.

Thus,the value of a European call option becomes

$$C(S,\ t) = \exp(-D_0\ (T-t))\ S\ N(d_{10}) - E\ \exp(-r\ (T-t))\ N(d_{20}),$$

where

$$d_{10} = \frac{\log(S/E) + \left(r - D_0 + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_{20} = d_1 - \sigma\sqrt{T-t}\ .$$

(d_1 is as previously defined).

Written by Alessio Farhadi

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