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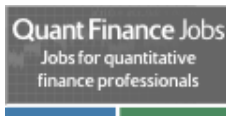
:: FUNDAMENTALS

:: Understanding the Central Limit Theorem ::

Consider a fair game of coin tossing. If a Tail (T) comes out you loose \$1 otherwise with a Head (H) you win \$1.

If we play twice then the list of possible outcomes are {TT,TH,HT,HH}. However the TH and HT have the same payoff and thus the payoff list for {TT,TH & HT,HH} is \$-2,0,2. Obviously here all possible outcomes are equally likely and so the possibility of coming out with nothing is twice as likely than winning or loosing.

Consider a second game with three outcomes Red (R), Green (G), Blue (B). With R you loose \$2; with B you win \$2; otherwise with G you gain nothing. If G is twice as likely to occur than the other 2 colors we immediately see that this game is equivalent to playing our first game twice. It has the same probabilities associated to the same possible payoffs.



Mathematically - the mapping of game 1 onto game 2 is done by an Auto or Self convolution of the probability distribution function of the payoffs of game 1.

:: Collaborations ::



AutoConvolution of {T,H}

$$= \{T_1, H_1\} \otimes \{T_2, H_2\} = \{T_1, \{T_2 H_1\}, H_2\} + \{\{T_1 T_2\}, \{H_1 H_2\}\} + \{T_2, \{T_1 H_2\}, H_1\} = \{T_2 H_1, T_1 T_2, H_1 H_2, T_1 H_2\}$$

Now the very important property that $T_2 H_1 =$

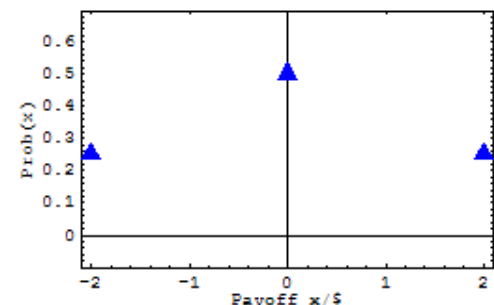
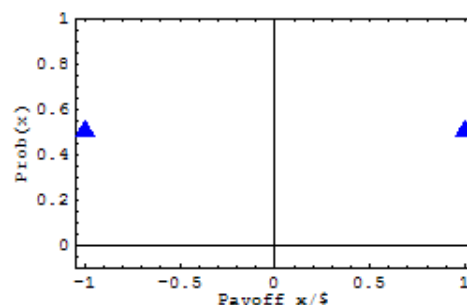
$T_1 H_2$ means that

AutoConvolution of {T, H} = {TT, 2 HT, HH}

The associated Probability distribution is

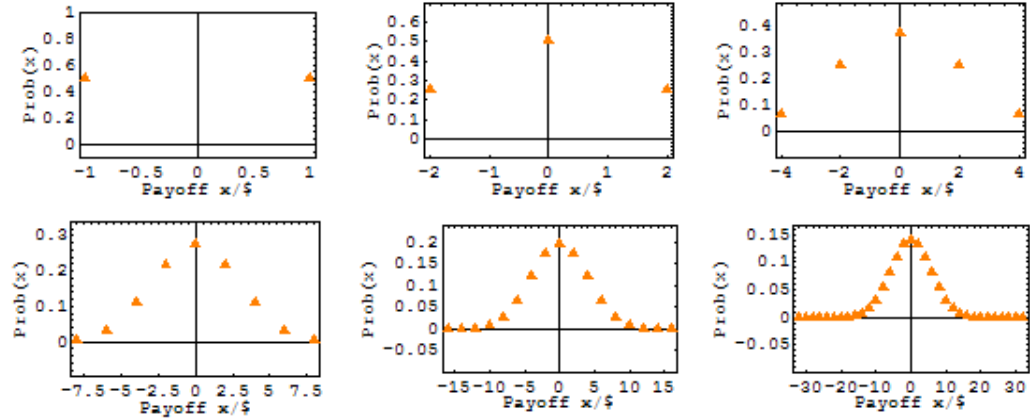
$$\{P(T), P(H)\} \otimes \{P(T), P(H)\} = \{P(TT), 2P(HT), P(HH)\} = \frac{1}{4} \{1, 2, 1\}$$

This is the same Probability distribution function for the payoff associated to {R,G,B} in the second game.



Playing Game 2 twice is equivalent to playing Game 1 four times etc. - and performing

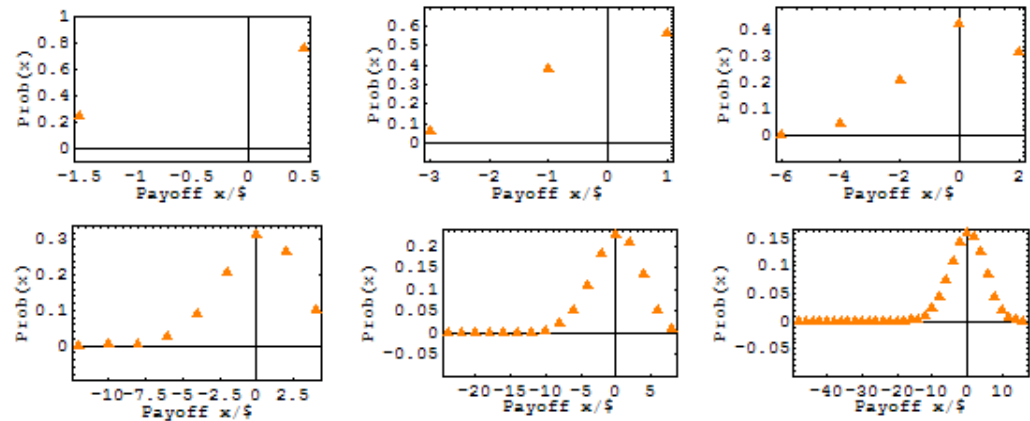
the Autoconvolution we can find the list of outcomes together with the probability distribution function.



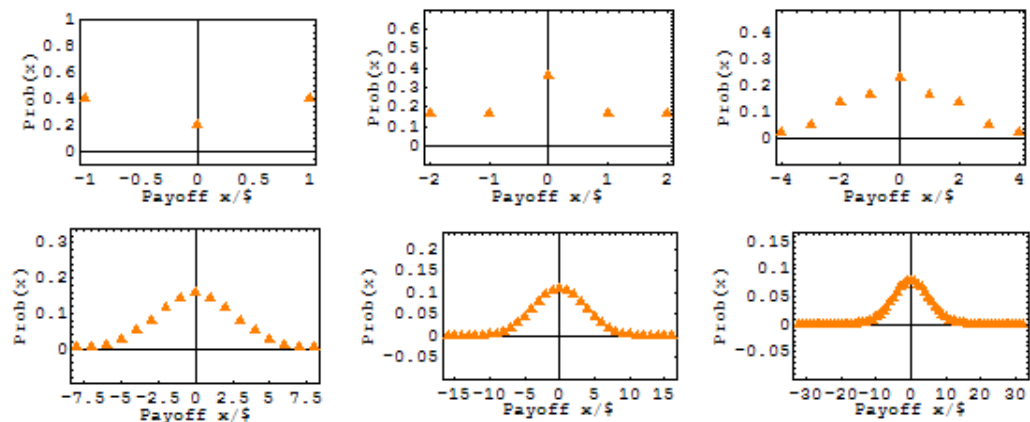
We observe that the Probability distribution function associated to the possible payoffs approaches a Gaussian distribution as we play more and more games.

This observation is part of what is known as the Central Limit Theorem.

We can play different fair H & T or RGB games with different associated probability distribution functions and still in the limit the probability of the payoff approaches a Gaussian.



Further we can think of fair RGB games that cannot be constructed by playing two different or same fair HT. For example the probabilities associated to {R,G,B} being {0.4,0.2,0.4} is one such example. Nevertheless, playing this game in repetition or as part of a series of different fair games still produces a Gaussian distribution.



Why? Why is it that when playing many fair games with different probability distribution functions associated to the payoffs do we always approach a Gaussian distribution?

The proof of this theorem requires graduate mathematics. However, we shall focus on the key property that is responsible for this.

Lets focus on H & T games and , for the moment, forget the payoff distribution function. Since the order of an outcome is not an issue here the ordered outcome TTHH gives the same payoff as THTH - the distribution of the outcomes is thus Binomial.

$$\begin{array}{l}
 T \quad H \\
 T^2 \quad 2HT \quad H^2 \\
 T^3 \quad 3HT^2 \quad 3H^2T \quad H^3 \\
 T^4 \quad 4HT^3 \quad 6H^2T^2 \quad 4H^3T \quad H^4 \\
 T^5 \quad 5HT^4 \quad 10H^2T^3 \quad 10H^3T^2 \quad 5H^4T \quad H^5 \\
 T^6 \quad 6HT^5 \quad 15H^2T^4 \quad 20H^3T^3 \quad 15H^4T^2 \quad 6H^5T \quad H^6
 \end{array}$$

Therefore the distribution of the outcomes approaches a Gaussian over many games. When we then compute the probability distribution for the payoff by substituting in $P(H)$ and $P(T)$ we see that any peak in the previous distribution will move towards the center driven by the increasing combinatorial paths leading towards the center.

For an RGB game the distribution is not the usual Binomial but still approaches a Gaussian since the order of individual outcomes does not matter. Indeed, the Pascal triangle structure obtained in the HT outcome distribution is also obtained in the RGB game with the difference that the former produces a Pascal triangle whilst the latter produces a Pascal tetrahedron. We just have to add the condition that RB produces the same payoff as G^2 - projecting the tetrahedron structure of the outcomes to a triangle.

$$\begin{array}{l}
 R \quad G \quad B \\
 R^2 \quad 2RG \quad G^2 + 2RB \quad 2GB \quad B^2
 \end{array}$$

Even when we play a series of many different fair H & T games with different probabilities associated to a H and T outcome - because H followed by T and T followed by H produce the same payoff - the outcome will always approach a Gaussian.

Written by Raffaello Vardavas. Special thanks to Cameron Connell.

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