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:: FUNDAMENTALS

:: The Greek Letters - Theta ::

The Greek Letters or simply the "Greeks" are quantities representing the market sensitivities of the options or other derivatives. Each Greek measures a different aspect of the risk in an option position. Through understanding and managing these Greeks, market makers, traders, financial institutions and portfolio managers can manage their risks appropriately, whether they deal in OTC or exchange-traded options. This article looks at the Greek that governs time decay of all options - Theta.

Theta

The theta of an option is defined as the rate of change of the option price w.r.t. decrease in time to maturity or expiry of the option, when all else remains the same. Theta is also referred to as the **time decay** of the option. Theta for an option can be obtained by simply partial differentiate option price/value with time:

$$\Theta = \frac{\partial V}{\partial t}$$

:: Collaborations ::



Derivation of Theta

Recalling that for a European call option on a dividend-paying share at rate q , the value of the call option is simply:

$$C(S, t) = e^{-q(T-t)} S N(d_1) - E e^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{\log(S/E) + (r - q + \frac{1}{2} \sigma^2)(T - t)}{\sigma \sqrt{T - t}}$$

and

$$d_2 = d_1 - \sigma \sqrt{T - t}$$

Therefore, theta (Θ) is simply

$$\begin{aligned} \Theta &= \frac{\partial V}{\partial t} \\ &= q e^{-q(T-t)} S N(d_1) + e^{-q(T-t)} S N'(d_1) \frac{\partial d_1}{\partial t} \\ &\quad - r E e^{-r(T-t)} N(d_2) - E e^{-r(T-t)} N'(d_2) \frac{\partial d_2}{\partial t} \end{aligned}$$

Now

$$\begin{aligned}
 \frac{\partial d_1}{\partial t} &= \frac{\log(S/E)}{2\sigma\sqrt{T-t}(T-t)} - \frac{(r - q + \frac{1}{2}\sigma^2)}{2\sigma\sqrt{T-t}} \\
 &= \frac{\log(S/E) - (r - q + \frac{1}{2}\sigma^2)}{2\sigma\sqrt{T-t}(T-t)} \\
 &= \frac{d_2}{2(T-t)} \\
 \frac{\partial d_2}{\partial t} &= \frac{\partial}{\partial t} (d_1 - \sigma\sqrt{T-t}) \\
 &= \frac{\partial d_1}{\partial t} + \frac{\sigma}{2(T-t)} \\
 &= \frac{d_2 + \sigma}{2(T-t)}
 \end{aligned}$$

Previously, from the **delta** article:

$$\begin{aligned}
 N'(x) &= \frac{1}{\sqrt{2\pi}} \exp(-x^2/2) \\
 N'(d_1) &= \frac{1}{\sqrt{2\pi}} \exp(-d_1^2/2) \\
 N'(d_2) &= \frac{1}{\sqrt{2\pi}} \exp(-d_2^2/2) \\
 &= \frac{1}{\sqrt{2\pi}} \exp(-d_1^2/2) \exp \\
 &\quad \left(\sigma\sqrt{T-t}d_1 - \sigma^2(T-t)/2 \right) \\
 &= \\
 &= N'(d_1) \exp \\
 &\quad \left(\log(S/E) + \left(r + \frac{1}{2}\sigma^2 \right) (T-t) - \right. \\
 &\quad \left. \sigma^2(T-t)/2 \right) \\
 &= N'(d_1) \frac{S}{E} e^{r(T-t)}
 \end{aligned}$$

Theta can now be presented in a simpler form:

$$\begin{aligned}
 \Theta &= q e^{-q(T-t)} S N(d_1) + e^{-q(T-t)} S N'(d_1) \frac{\partial d_1}{\partial t} \\
 &\quad - r E e^{-r(T-t)} N(d_2) - E e^{-r(T-t)} N'(d_2) \frac{\partial d_2}{\partial t} \\
 &= q e^{-q(T-t)} S N(d_1) - r E e^{-r(T-t)} N(d_2) \\
 &\quad + e^{-q(T-t)} S N'(d_1) \frac{d_2}{2(T-t)} - \\
 &\quad E e^{-r(T-t)} \left(N'(d_1) \frac{S}{E} e^{r(T-t)} \right) \frac{d_2 + \sigma}{2(T-t)} \\
 &= q e^{-q(T-t)} S N(d_1) - r E e^{-r(T-t)} N(d_2) - \\
 &\quad \frac{S N'(d_1) \sigma}{2\sqrt{T-t}}
 \end{aligned}$$

Therefore, for a European call option on a dividend-paying (at rate q) share, theta is simply:

$$\Theta_{\text{call}} = -\frac{S N'(d_1) \sigma}{2 \sqrt{T-t}} + q S N(d_1) e^{-qT} - r E e^{-rT} N(d_2)$$

Similarly, for a European put option of the same underlying, theta is given by:

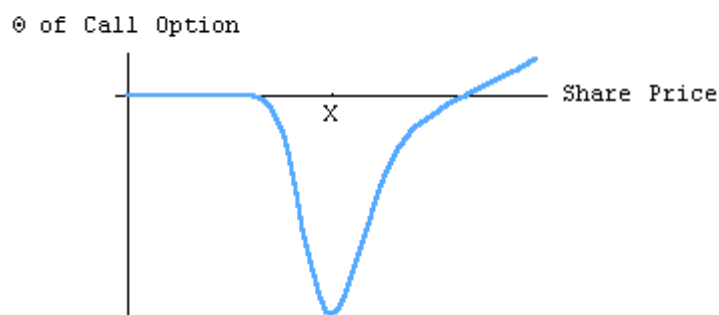
$$\Theta_{\text{put}} = -\frac{S N'(d_1) \sigma}{2 \sqrt{T-t}} - q S N(-d_1) e^{-qT} + r X e^{-rT} N(-d_2)$$

Since T is measured in years, theta is also measured in years. To obtain theta measured in days, simply divide theta from the formulae by the number of (trading) day.

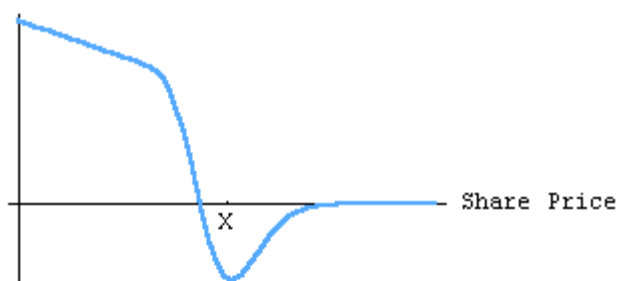
As time passes, an option will lose its time value and theta measures the extent of this decay. Since both call and put options are wasting assets, theta is usually **negative** for an option, except in-the-money European put option on a non-dividend-paying share or in-the-money European call option with high dividend rate.

Variation of Theta with Share Price

Variation of theta with share price (S) for a European option on a non-dividend-paying share with exercise/strike price of E. Here one can see that theta approaches **zero** for **out-of-the-money** options, **large and negative** for **at-the-money** options and tends to $rX e^{-rT}$ for **in-the-money** options. Here one can see that theta is **positive** for **deep in-the-money** options, since the options are for certain going to be exercised and hence their values increase with favourable share price movement.



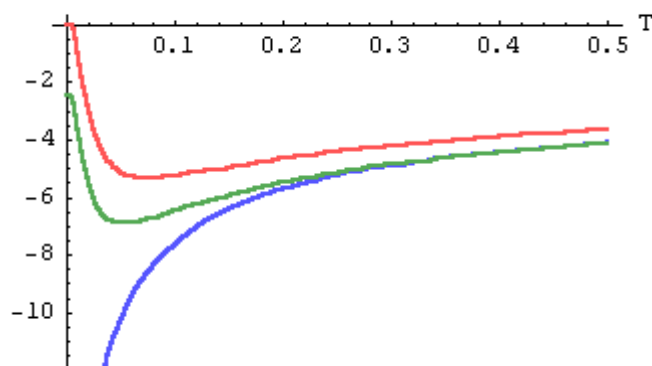
Θ of Put Option



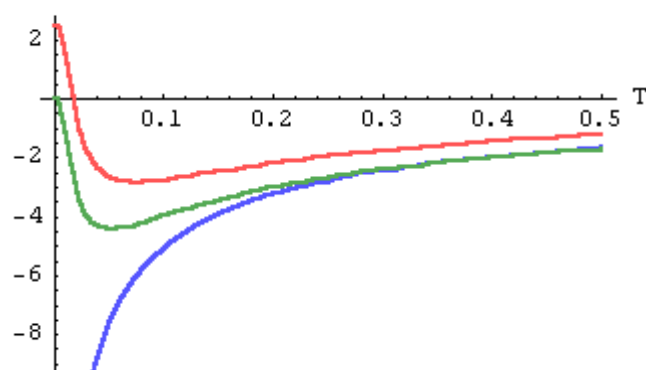
Variation of Theta with Time to Expiry

Variation of theta with Time to Expiry (T) for European option on a non-dividend-paying share with exercise price of E. **Red**, **Blue** and **Green** lines denote out-of-the-money, at-the-money and in-the-money options respectively. (Values used: $S = £47.5$, $£50$, $£52.5$, $E = £50$, $\text{vol} = 20\%$, $r = 5\%$ and $q = 0\%$).

Θ for Call Option



Θ for Put Option



Written by Henry Tang.

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