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:: FUNDAMENTALS

:: Interest Rate Modelling 1 - Introduction ::

This is the first of a series of articles that will tackle interest rate modelling in finance. Here, we introduce the important concepts relevant to the fixed income world, while later articles will emphasize more on the models that are available and their relative strengths and weaknesses.

In the deterministic world, interest rate is assumed to be fully defined. This is very convenient, because then expressions such as

$$B(t) = \exp \left(\int_0^t r(s) ds \right)$$

(which characterizes the well know 'money market' or bank account) have an intuitive meaning. The expression dictates how a unit of currency invested in a bank today will grow over any time horizon $[0, T]$. Likewise, the value of a (default risk free) bond can thus be determined as the discounted future cash flows of the coupons and the notional; this discounting being applied using the deterministic interest rate. This is a result of simple arbitrage arguments: if the price of the bond was less than the discounted value then a strategy which buys the bond (financing this purchase with a borrowing at the risk free rate) will generate a risk free profit. Another consequence of a deterministic interest rate is that it gives investors a deterministic outlook on future borrowing or saving. So, for e.g., an investor today will know with 100% certainty what interest rate he/she will be faced with in the future, and will know today exactly how much they will need to invest in one years time for it to grow to a specified value X by the second year. Moreover, this outlook DOES NOT CHANGE at any later date. Deterministic interest rate means that everything relating to investing and borrowing cash is, and stays, well defined and static.

This is convenient but not realistic for longer time horizons. Interest rates do change and bond prices do not grow smoothly and deterministically. The chart below illustrates the historical 5 year yield on UK risk free government bonds. Since the bonds are risk free, the yield is essentially closely associated with the risk free interest rate (the yield is the 'average' interest rate over the next 5 year window period, [see earlier article](#)). The fact that these yields are not static implies non constant interest rates. Furthermore, there is nothing, whether in the finance or in the mathematics, to suggest that the yield changes can be predicted (you can at best make an estimate based on some economic study, but this is still uncertain). Hence the need to model interest rates stochastically.

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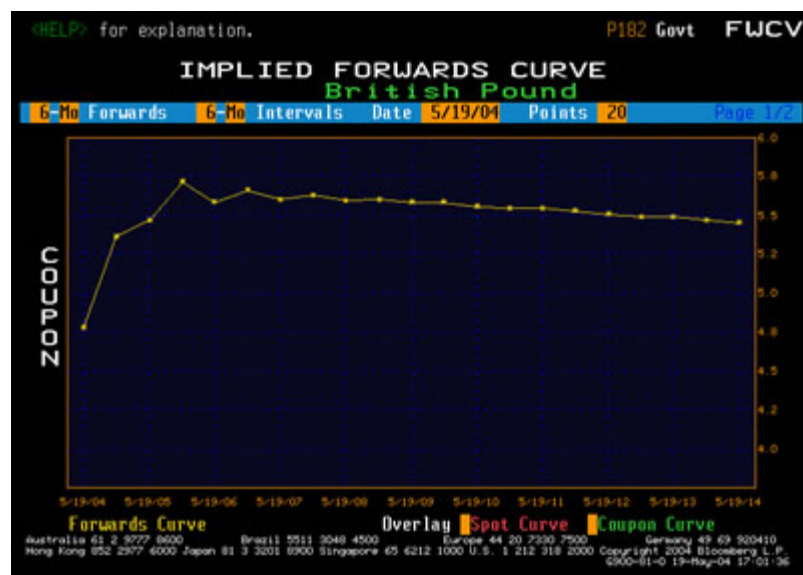




When you decide that you want to worry about random interest rates, fixed income becomes a more complicated subject. Expressions like the one above become less intuitive. $\text{Exp}\left(\int_0^T r(s) ds\right)$, which still characterises the growth of money in the money markets account, is now random. In later articles, we show that we can no longer deduce deterministically the growth of money, but instead, only attach a probability distribution to it. But this begs the question as to how bonds can be priced.

Another repercussion of stochastic interest rate modelling is that it no longer offers investors the knowledge of future borrowing/investing of cash. This can now only be implied and can change day on day depending on the market prices of bonds. The implied future interest rate is called the forward rate and is basically the interest rate that can be generated or locked into by taking positions in bonds. For example, an investor who knows that he/she will need to borrow at the end of year one, with repayment at the end of the second year can take a long position in the zero coupon bond $B(t,1)$ and a short position in the $B(t,2)$ zero coupon bond. If the position in the second bond is a short position of $B(t,1)/B(t,2)$ then the value of the $B(t,1)$ position exactly balances the $B(t,2)$ position so that it is costless to enter into (ignoring transaction costs). At the end of year one, the investor receives the face value of the year 1 maturity bond (say \$1); at the end of year 2 the investor MUST (because they are short the bond) pay out the face value of $B(t,1)/B(t,2)$ year 2 maturity bonds. This is akin to borrowing at the end of year 1 with a repayment at the end of year 2. The interest paid is $B(t,1)/B(t,2)-1$. Note that this interest rate is fixed at the initial time despite the fact that interest rate is stochastic, but will change as soon as the bond prices shift. Hence, the future outlook on borrowing and lending is now no longer fixed. The implied interest rate is referred to as the forward rate and the forward 'curve' characterizes future interest rates using ALL the default risk free bonds available in the market. So we may have the 3 month rate (borrowing today with repayment in 3 months), 3-6 month rate (borrowing in 3 months time, repayment in 6 months time), 6-12 month rate, etc. For e.g. the curve below is the six-monthly forward rates obtained from the UK Gilts market. The first point is usually the spot interest rate as dictated by the market at the time; successive points

describe the interest rate that can be locked into for six month borrowing/lending periods using the UK Gilts traded at the time.



The curve is an important piece of information for some of the more advanced interest rate modelling techniques which will be addressed in future articles (e.g. HJM, BGM). Clearly, forward rates can only be derived if bonds can be priced, and bonds are the most basic 'interest rate derivatives' to price, which brings us to the core problem. We have already stated that the expression $\exp\left(\int_0^T r(s) ds\right)$ no longer works since it is not a defined quantity so other means must be found. Furthermore, what about Bond options, Caps, Floors, etc? No mention of these has yet been made, although they are fixed income securities which frequently need to be priced. Since interest rates become random, so do bond prices (look at this article for the relationship between yield/bond price). Once we are capable of pricing bonds, we can describe their random evolution as well, and then we can go on to price options on these bonds (like pricing options on stocks). Needless to say, there is a huge selection of texts that take you through the hedging arguments that allow you to price interest rate derivatives. None of this mathematics will be discussed in this series of articles; instead we focus on the logic and intuition behind the maths which we feel is weakly conveyed in the texts.

There are essentially 3 generations of interest modelling derivatives. The first generation models proposed to model the interest rate directly; they are the so called short rate models. By short rate here, we mean the short term instantaneous interest rate that is observed in continuous time (borrowing now and repayment after a small time interval). Second generation models comprise the Heath-Jarrow-Morton models which attempt to model the entire forward curve described above. Since the curve is closely associated with bond prices, bond price dynamics can be inferred from it. The advantage of the HJM type approach over the short rate approach is that it achieves an automatic fit to the yield curve, where the short rate models require some extra computation. This fitting or 'calibration' will be re-addressed in later articles. The latest generation of models (LIBOR Market Model/Brace-Gatarek-Musiela) attempt to model specific parts of the forward curve. Essentially, they take the HJM models from being largely theoretical to more practical and applicable but

looking at the forward curve at only a discrete set of points.

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