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:: FUNDAMENTALS

Options for Discrete Dividend Paying Assets

Previously we covered idealised assets with continuous dividend yields. In reality, all dividend payments are paid out discretely. Furthermore, there may be very few dividend payments that one may receive during the ownership of an asset to permit the description of payments as continuous. Following from our previous article on dividend payments, we now consider discrete dividend payment and the impact they have on option prices

As already discussed, a dividend payment is a fraction δ of the asset value S that is paid to the owner of the asset. Let t denote the time of dividend payment (in continuous time), therefore, at time $t - \delta t$ just before the asset is priced at $S(t - \delta t)$, and $S(t + \delta t)$ at time $t + \delta t$ just after. We know that

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We can relate these two prices to the dividend payment δ by

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From this expression we see a discontinuity in the price of the asset which we need to incorporate into the pricing of an option. We have to remember that the owner of the option receives no dividend payments (it is the owner of the asset), however, the drop in price is important to account for. If we do not account for the dividend payment, we may over price the option since the asset value drops and we may fall below our exercise price.

We also require the option value to be continuous over the dividend payment period to avoid any arbitrage opportunities occurring. To satisfy this condition we require the value of the option $V(S,t)$ to satisfy

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or

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So, the asset price changes discontinuously over the period of dividend payment, however, the option price does not.

We need to somehow implement the jump condition at the time of dividend payment t .

Let us consider a European call option C on an asset that pays

only one dividend payment (one *jump condition*) till maturity time T . We can break the life of the option into two periods, after the dividend payment till maturity ($T > t > t_d$), and, from the present time till the dividend payment date ($t_d > t > 0$). One can solve the Black-

Scholes for each of these periods provided we implement the jump condition. Since the Black-Scholes is solved backwards from the dividend maturity date (backward parabolic). After the dividend payment we can solve the Black-Scholes as normal since no further dividends will be paid $C(S, t; E)$ with exercise price E .

Hence,



Using the jump condition one has



Therefore, by solving the Black-Scholes equation for and implementing the jump condition we can find the price of the call option before the dividend payment.

It so happens (you can show this!) that by transforming the Black-Scholes for , our call option value before the dividend payment

is similar to solving for calls with exercise price

. Thus,



Now, we have accounted for the jump condition due to the discrete dividend payment of the asset which have a net reduction in the pricing of the European call option (also true for other types of options).

As we have already stated in the previous article on dividend paying assets, the forecasting of dividend yields is an art in itself critical to derivative pricing. Many derivative teams employ the services of financial analyst to help them with this the forecasting.

Mathematical Side Note:

The asset price for discrete dividend payments can be written as



which can be integrated across the dividend date to give



As , only the term with the delta function survives. Therefore, we are left with

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Integrating the delta function over this range gives us the heavy side function  rGranh, thus,

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discounted.

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